Traitement des signaux bivariés: de nouveaux outils pour l'acoustique sous-marine

Workhop SERENADE

Julien Flamant

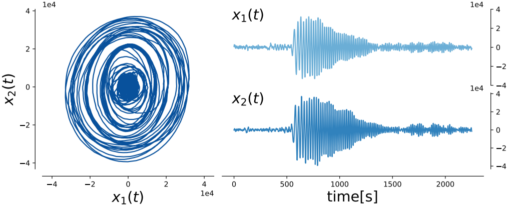
CNRS, CRAN, Université de Lorraine

Brest, 29 juin 2022

joint work with J. Bonnel, N. Le Bihan, P. Chainais

Bivariate signals in nature

polarized
seismic waves



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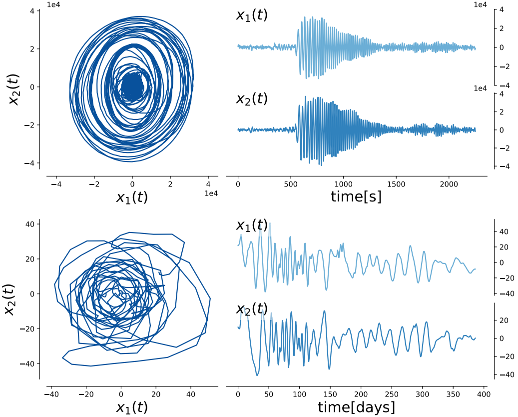

Bivariate signals in nature

polarized
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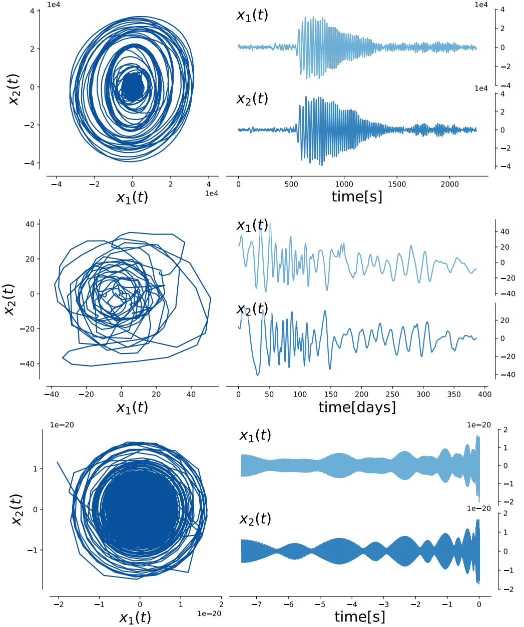
gravitational waves in
precessing binaries



Particle velocity in underwater acoustics

Linearized Euler equation

$$\nabla p = -\rho \frac{\partial \mathbf{v}}{\partial t}$$

- pressure p
- particle velocity $\mathbf{v}$
- ocean density ρ

assuming cylindrical symmetry:
$\mathbf{v} = [v_r, v_z] \longrightarrow$ bivariate signal !

$v_r(t)$: radial component
$v_z(t)$: vertical component

IVAR recorder [Dahl and Dall'Osto, 2019]

Particle velocity in ecology too!

Methods in Ecology and Evolution

Methods in Ecology and Evolution 2016, **7**, 836–842 doi: 10.1111/2041-210X.12544

Particle motion: the missing link in underwater acoustic ecology

Sophie L. Nedelec[1]*, James Campbell[2], Andrew N. Radford[1], Stephen D. Simpson[3] and Nathan D. Merchant[4]

[1]School of Biological Sciences, Life Sciences Building, University of Bristol 24 Tyndall Avenue, Bristol, BS8 1TQ, UK; [2]Behavioral Biology, Institute of Biology (IBL), Leiden University, The Netherlands; [3]Biosciences, College of Life and Environmental Sciences, Geoffrey Pope, Stocker Road, Exeter, EX4 4QD, UK; and [4]Centre for Environment, Fisheries & Aquaculture Science (Cefas), Lowestoft, Suffolk, NR33 0HT, UK

The importance of particle motion to fishes and invertebrates

JASA (2018)

Arthur N. Popper[1,a)] and Anthony D. Hawkins[2]

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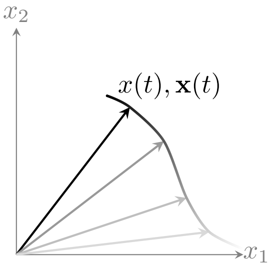
This paper considers the importance of particle motion to fishes and invertebrates and the steps that need to be taken to improve knowledge of its effects. It is aimed at scientists investigating the impacts of sounds on fishes and invertebrates but it is also relevant to regulators, those preparing environmental impact assessments, and to industries creating underwater sounds. The overall aim

Different but equivalent representations

vector $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ complex $x(t) = x_1(t) + \boldsymbol{i}x_2(t)$



bivariate signal processing

$\longleftrightarrow$

tools for the joint analysis / processing of 2 components $x_1(t)$ and $x_2(t)$

Vector representation of bivariate signals

$$\boxed{\mathbf{x}(t) = [x_1(t), x_2(t)]^\top \in \mathbb{R}^2}$$

- Special case of analysis of multivariate vector signals

  Hannan (1970), Priestley (1981)

- Polarization in optics

  Born and Wolf (1980), Goodman (1984), Mandel and Wolf (1995)

- Jones matrix-vector calculus

  Jones (1941), Azzam and Bashara (1978)

- Instantaneous polarization attributes in seismology

  Diallo et al. (2005), Roueff et al. (2006)

Complex representation of bivariate signals

$$\boxed{x(t) = x_1(t) + \boldsymbol{i}x_2(t) \in \mathbb{C}}$$

- Circularity of random complex signals (rotational invariance)
  Picinbono (1994), Amblard et al. (1996)
- Augmented representations ${}_{\uparrow}\mathbf{x}(t) = [x(t), \overline{x(t)}]^\top$
  Schreier and Scharf (2003, 2010), Adalı et al. (2011)
- Rotary components $x \equiv \sum \circlearrowleft + \sum \circlearrowright$
  Blanc-Lapierre and Fortet (1953), Gonella (1972), Walden (2013)
- Instantaneous ellipse description for nonstationary bivariate signals
  deterministic Lilly and Olhede (2010) random Schreier (2008)

  EMD Rilling et al. (2007)

The need for interpretability

existing approaches: no straightforward physical descriptions

feature	$\mathbf{x}(t) \in \mathbb{R}^2$	$x(t) \in \mathbb{C}$	desired
direct ellipse parametrization	✗	✗	✓
positive frequencies	✓	✗	✓
interpretable filtering relations	✗	✗	✓

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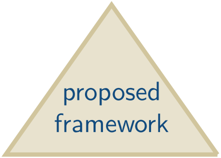

$\Downarrow$

efficient, relevant generalization of ubiquitous signal processing tools

The need for interpretability

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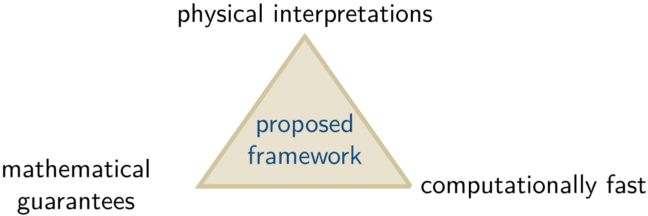
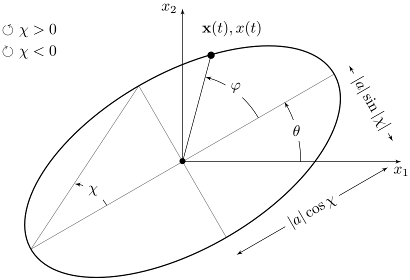


$\Downarrow$

efficient, relevant generalization of ubiquitous signal processing tools

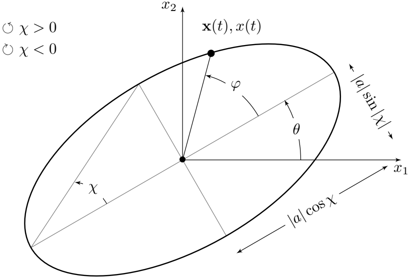
Monochromatic bivariate signal representation



Polarization ellipse parameters

- $a \geq 0$ intensity
- $\theta \in [-\pi/2, \pi/2]$ orientation
- $\chi \in [-\pi/4, \pi/4]$ ellipticity
- $\varphi \in [0, 2\pi)$ phase

Monochromatic bivariate signal representation



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Jones vector: Vector representation (optics, seismology)

$$\mathbf{x}(t) = \begin{bmatrix} A_1 \cos(2\pi f_0 t + \Phi_1) \\ A_2 \cos(2\pi f_0 t + \Phi_2) \end{bmatrix} \quad \theta, \chi \leftarrow f(A_1, A_2, \Phi_1, \Phi_2)$$

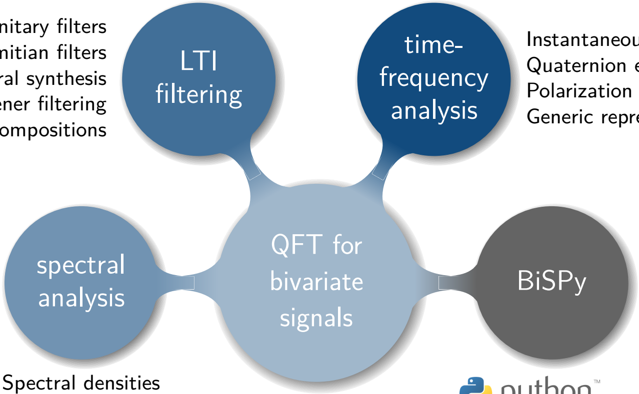
Rotary components: Complex representation (oceanography, signal processing)

$$x(t) = A_+ e^{\boldsymbol{i}\theta_+} e^{\boldsymbol{i}2\pi f_0 t} \qquad \theta \leftarrow g_1(\theta_+, \theta_-)$$

$$+ A_- e^{-\boldsymbol{i}\theta_-} e^{-\boldsymbol{i}2\pi f_0 t} \qquad \chi \leftarrow g_2(A_+, A_-)$$

A general framework for bivariate signal processing

Unitary filters
Hermitian filters
Spectral synthesis
Wiener filtering
Original decompositions

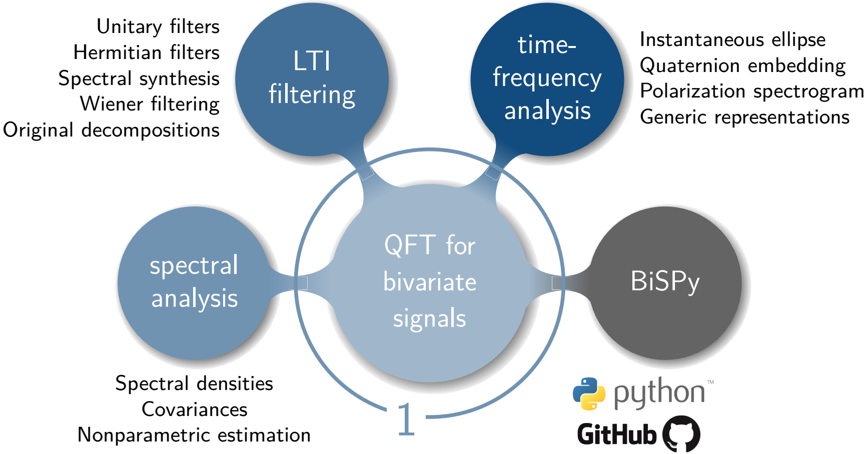


Instantaneous ellipse
Quaternion embedding
Polarization spectrogram
Generic representations

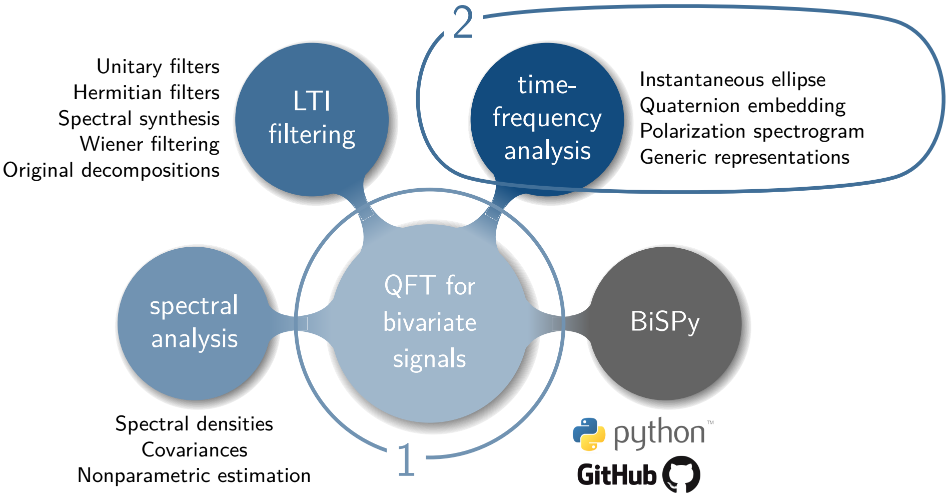
Spectral densities
Covariances
Nonparametric estimation

python™
GitHub

A general framework for bivariate signal processing



A general framework for bivariate signal processing



Quaternion Fourier transform for bivariate signals

Framework

Ingredient #1 bivariate signal as complex signal embedded in $\mathbb{H}$

$$x(t) = x_1(t) + \boldsymbol{i}x_2(t) \in \mathbb{C}_{\boldsymbol{i}} \subset \mathbb{H}$$

alike embedding a signal $x(t) \in \mathbb{R}$ into $\mathbb{C}$

Quaternions

4D algebra $\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = -1$ ⚠ $\boldsymbol{ij} = \boldsymbol{k}$, $\boldsymbol{ij} = -\boldsymbol{ji}$ ⚠

complex subfields of $\mathbb{H}$: $\mathbb{C}_{\boldsymbol{i}} = \mathrm{Span}\,\{1, \boldsymbol{i}\}$, $\mathbb{C}_{\boldsymbol{j}} = \mathrm{Span}\,\{1, \boldsymbol{j}\}$, ...

polar forms, 3D and 4D geometry, etc.

Framework

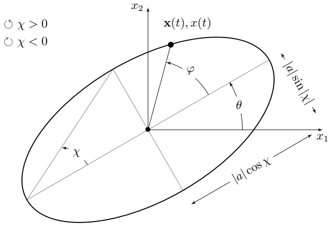
Ingredient #2 adapt Fourier transform

Quaternion Fourier Transform (QFT)

$$X(f) = \int \underbrace{x(t)}_{\in \mathbb{C}_{\boldsymbol{i}}} \underbrace{e^{-\boldsymbol{j}2\pi ft}}_{\in \mathbb{C}_{\boldsymbol{j}}} \mathrm{d}t \in \mathbb{H}$$

Monochromatic polarized signal

polar form by Bülow and Sommer (2001)



$$x(t) = \mathrm{Proj}_{\mathbb{C}_{\boldsymbol{i}}}\left\{ a e^{\boldsymbol{i}\theta} e^{-\boldsymbol{k}\chi} e^{\boldsymbol{j}(2\pi f_0 t + \varphi)} \right\}$$

$\updownarrow$ QFT

$$X(f) = a e^{\boldsymbol{i}\theta} e^{-\boldsymbol{k}\chi} e^{\boldsymbol{j}\varphi} \delta_{f_0}(f) + \text{sym.}$$

polar form $\leftrightarrow$ physical parameters

QFT properties

Existence for L^1, L^2 functions Jamison Ph.D. thesis (1970)
Easy to compute

$$x(t) = x_1(t) + \boldsymbol{i} x_2(t) \xleftrightarrow{\text{QFT}} X(f) = \underbrace{X_1(f)}_{1,\boldsymbol{j}} + \underbrace{\boldsymbol{i} X_2(f)}_{\boldsymbol{i},\boldsymbol{k}}$$

For bivariate signals keep $f \geq 0$ only (*$\boldsymbol{i}$-Hermitian symmetry*)

$$X(-f) = -\boldsymbol{i} X(f) \boldsymbol{i}, \quad \text{for } x(t) \in \mathbb{C}_{\boldsymbol{i}}$$

2 invariants for finite energy signals (QFT Parseval theorem)

$$\int_{-\infty}^{+\infty} |x(t)|^2 \,\mathrm{d}t = \int_{-\infty}^{+\infty} |X(f)|^2 \,\mathrm{d}f \qquad \text{(energy)}$$

$$\boxed{\int_{-\infty}^{+\infty} x(t) \boldsymbol{j} \overline{x(t)} \,\mathrm{d}t = \int_{-\infty}^{+\infty} \underbrace{X(f) \boldsymbol{j} \overline{X(f)}}_{\in \operatorname{span}\{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}} \,\mathrm{d}f \qquad \text{(geometry)}}$$

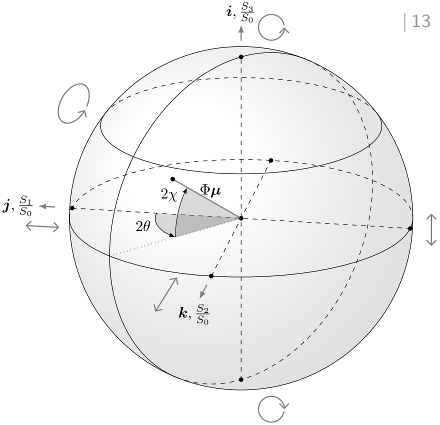
[Flamant, Le Bihan, Chainais, 2017]

Stokes parameters

Definition [Born and Wolf, 1980]

$$S_0 = a^2$$
$$S_1 = a^2\Phi \cos 2\theta \cos 2\chi$$
$$S_2 = a^2\Phi \sin 2\theta \cos 2\chi$$
$$S_3 = a^2\Phi \sin 2\chi$$

standard description of polarization properties in optics



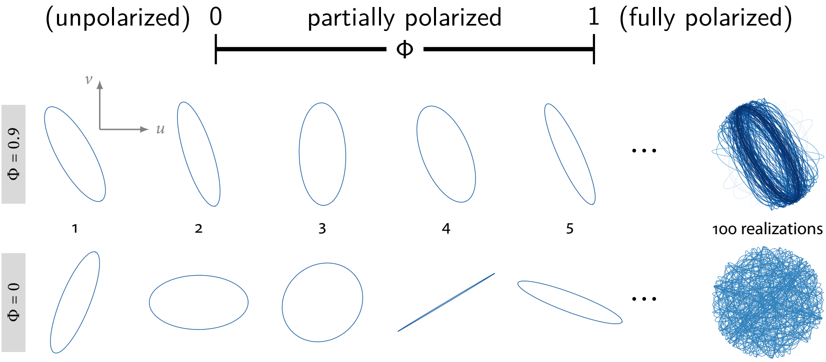
Poincaré sphere

Basic quaternion calculus shows that:

$$|X(f)|^2 = S_0(f), \quad X(f)\boldsymbol{j}\overline{X(f)} = \boldsymbol{i}S_3(f) + \boldsymbol{j}S_1(f) + \boldsymbol{k}S_2(f)$$

QFT Parseval invariants $\longleftrightarrow$ Stokes parameters $\longleftrightarrow$ spectral densities

Degree of polarization Φ



the degree of polarization Φ quantifies the statistical dispersion of ellipses

- deterministic signals $\Phi(f) = 1, \forall f$
- random signals $\Phi(f) \in [0, 1]$

What about underwater acoustics ?

Stokes parameters of normal modes

low-frequency acoustics in shallow water = normal modes

setting: broadband source $\Omega(f)$ at depth z_s, receiver at range r, depth z

$$p(f, r, z) \propto \Omega(f) \sum_{n=1}^{M} \Psi_n(f, z_s) \Psi_n(f, z) \frac{e^{-\boldsymbol{j} r k_n(f)}}{\sqrt{k_n(f) r}}$$

- $\Psi_n(f, z)$: modal depth functions of the waveguide
- $k_n(f)$: complex wavenumber

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- $k_n(f)$: complex wavenumber

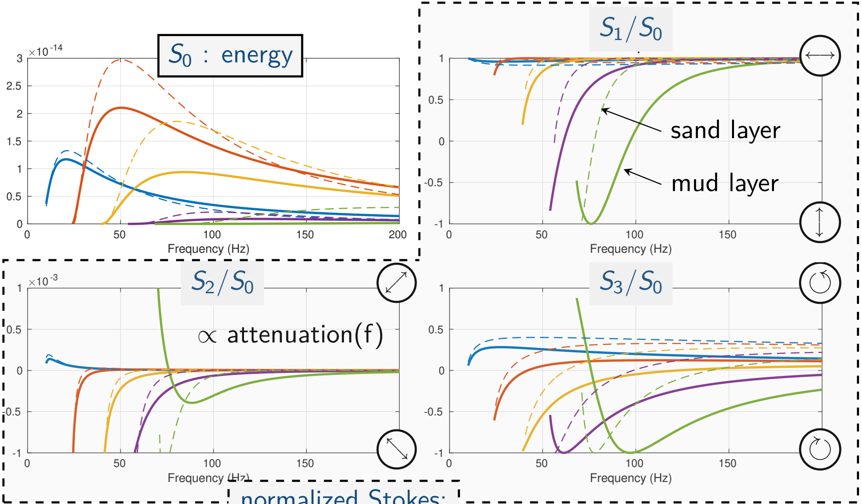
write the pressure associated at mode n as $p_n(f) = A_n(f)\Psi_n(f,z)$

$$V_{r_n}(f) \approx \frac{A_n(f)}{2\pi f\rho}k_n(f)\Psi_n(f,z) \qquad V_{z_n}(f) = \boldsymbol{j}\frac{A_n(f)}{2\pi f\rho}\frac{\partial\Psi_n(f,z)}{\partial z}$$

constructing the bivariate signal $V_n(f) = V_{r_n}(f) + \boldsymbol{i}V_{z_n}(f)$
$\longrightarrow$ explicit expressions for the Stokes parameters of particle velocity

Stokes parameters of normal modes

simulated 2-layer seabed inspired by SBCEX17 (semi ∞ basement + layer)



normalized Stokes: polarization

Stokes parameters of normal modes

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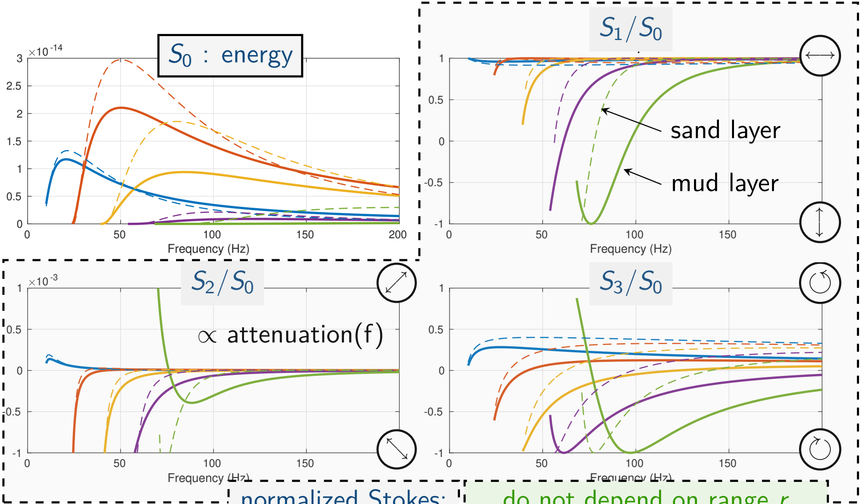
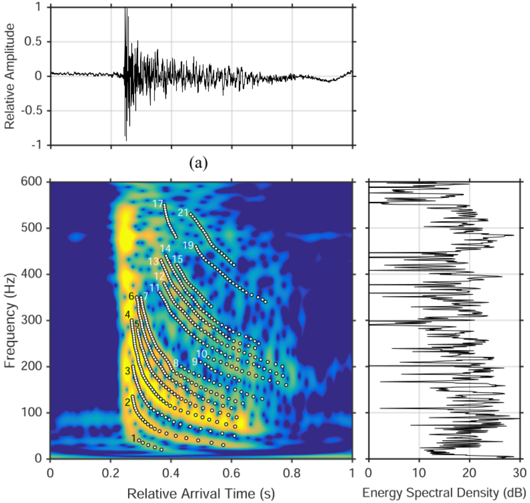


normalized Stokes: polarization

do not depend on range r and source depth z_s!

Time-frequency analysis of bivariate signals

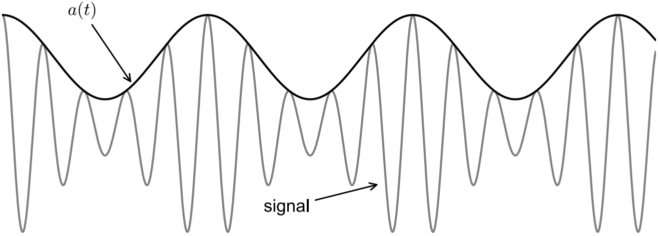


fundamental property: Hermitian symmetry of FT of real signals

Analytic signal of a real signal — Gabor (1946), Ville (1948)

One to one corresp. between a real signal and its analytic signal

$$x(t) \in \mathbb{R} \longleftrightarrow x_+(t) \in \mathbb{C}$$
$$a(t)\cos[\varphi(t)] \longleftrightarrow a(t)e^{i\varphi(t)}$$



Amplitude Modulation

Quaternion embedding of bivariate signals

$x(t) = x_1(t) + \boldsymbol{i} x_2(t) \qquad X(-f) = -\boldsymbol{i} X(f) \boldsymbol{i} \quad$ ($\boldsymbol{i}$-Hermitian symmetry)

Quaternion embedding

One-to-one correspondence

$$\text{bivariate signal} \longleftrightarrow \text{quaternion embedding}$$
$$x(t) \in \mathbb{C}_{\boldsymbol{i}} \longleftrightarrow x_+(t) \in \mathbb{H}$$

Polar form: instantaneous attributes

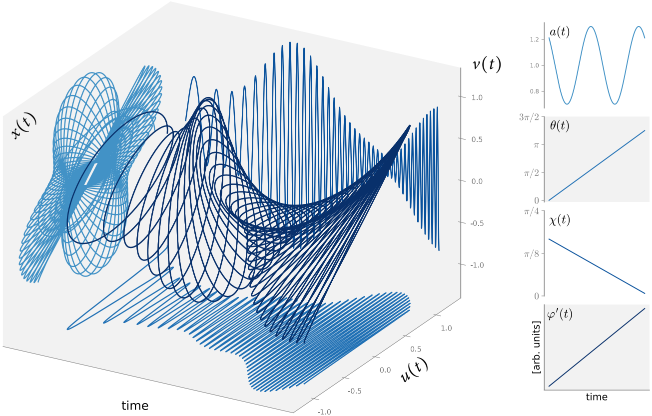
$$x_+(t) = \underbrace{a(t)}_{\text{amplitude}} \times \underbrace{e^{\boldsymbol{i}\theta(t)} e^{-\boldsymbol{k}\chi(t)}}_{\text{geometry}} \times \underbrace{e^{\boldsymbol{j}\varphi(t)}}_{\text{phase}}$$

$$a(t) \geq 0$$
$$\theta(t) \in [-\pi/2, \pi/2]$$
$$\chi(t) \in [-\pi/4, \pi/4]$$
$$\varphi(t) \in [-\pi, \pi]$$

Canonical quadruplet

Bivariate AM-FM signal model

$$x(t) = \mathrm{Proj}_{\mathbb{C}_{\boldsymbol{i}}}\{x_+(t)\} = a(t)e^{\boldsymbol{i}\theta(t)}\left[\cos\chi(t)\cos\varphi(t) + \boldsymbol{i}\sin\chi(t)\sin\varphi(t)\right]$$



bivariate linear chirp w/ amplitude, orientation and ellipticity modulation

justifies a posteriori Lilly and Olhede's (2010) parametric model

Polarization spectrogram

Quaternion Short Term Fourier Transform

Extend the STFT to the QFT setting

$$\mathrm{F}_x^g(\tau, f) = \int \underbrace{x(t)}_{\in \mathbb{C}_{\boldsymbol{i}}} \underbrace{g(t-\tau)}_{\in \mathbb{R}} \underbrace{\exp(-\boldsymbol{j} 2\pi f t)}_{\in \mathbb{C}_{\boldsymbol{j}}} \mathrm{d}t$$

$$\text{Theorems} \quad \begin{cases} \text{inversion} \\ \text{conservation: energy} \quad \text{geometry/polarization} \end{cases}$$

$|\mathrm{F}_x^g(\tau, f)|^2 \rightarrow$ Time-frequency energy density (S_0)

$\mathrm{F}_x^g(\tau, f)\boldsymbol{j}\overline{\mathrm{F}_x^g(\tau, f)} \rightarrow$ Time-frequency-polarization features (S_1, S_2, S_3)

new interpretable time-frequency-polarization representation

What about underwater acoustics ?

Polarization of normal modes in underwater acoustics

IVAR data from the SBCEX17 experiment

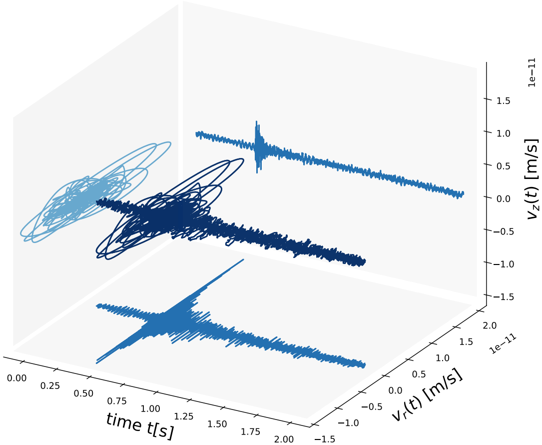
impulsive source $z_s \approx 20$ m, $r \approx 16$ km

IVAR detector
$z \approx 1$ m above seafloor

preprocessing
geometric projection

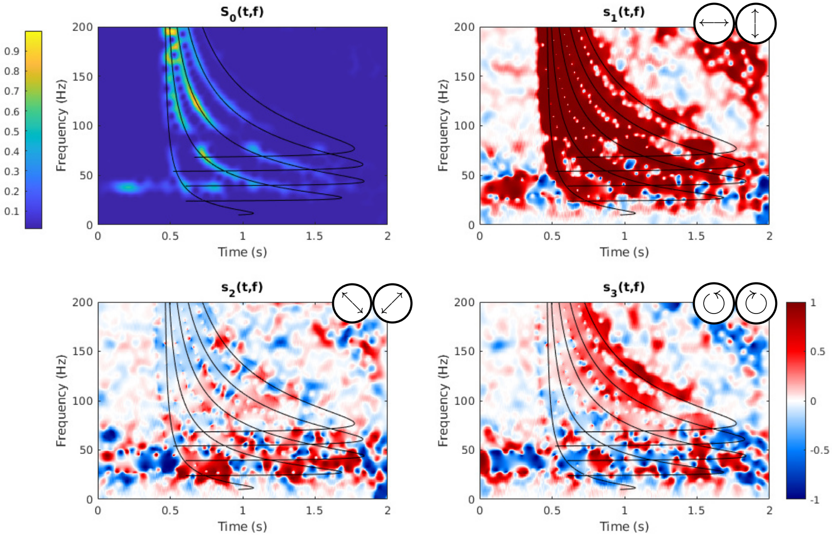
$$[v_x, v_y] \to v_r$$

+ bandpass filtering
+ source deconvolution



use the polarization spectrogram to reveal normal modes !

Polarization spectrogram of normal modes

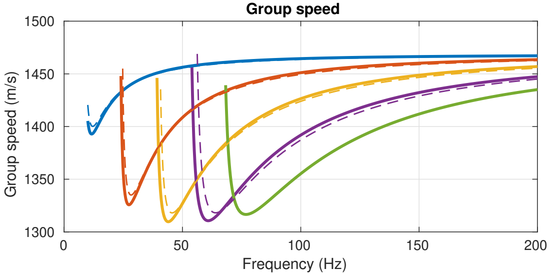


mode polarization characterizes of the environement

More polarization in underwater acoustics

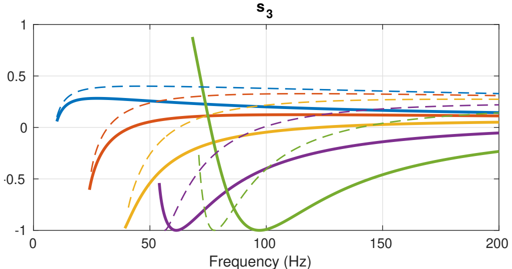
Stokes parameters: new observables for underwater acoustics

back to SBCEX17 model: mud layer (——) sand layer (- - -)



derived from
pressure measurement

limited sensitivity



derived from
particle velocity

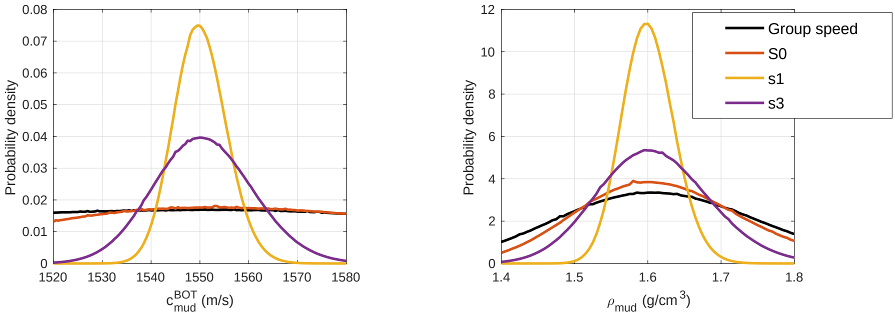
high sensitivity

Stokes parameters: geoacoustic inversion

Bayesian framework

- data: noisy modal metrics (Stokes + group speed), Gaussian noise with known covariance matrix
- each seabed parameter is inverted individually
- uniform prior

Posterior distributions



Underwater ecology with polarization

from light polarization ...

Circularly Polarized Light as a Communication Signal in Mantis Shrimps

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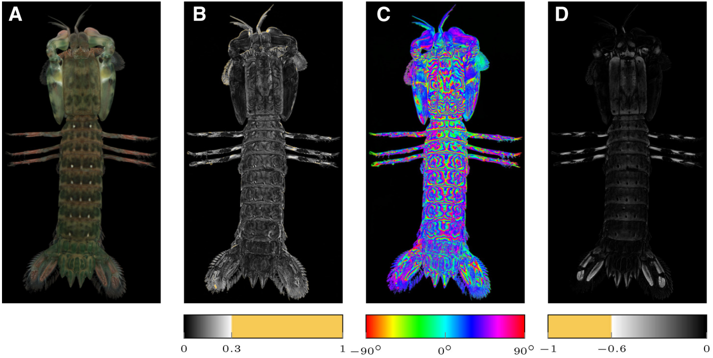
[2]School of Biological Sciences, University of Bristol, 24 Tyndall Avenue, Bristol BS8 1TQ, UK



RGB DoLP AoP LCP

Underwater ecology with polarization

... to particle motion polarization

North Sea soundscapes from a fish perspective: Directional patterns in particle motion and masking potential from anthropogenic noise

Peter Rogers,[1] Elisabeth Debusschere,[2] Dick de Haan,[3] Bruce Martin,[4,a)] and Hans Slabbekoorn[5,b)]
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[2]*InnovOcean Site, Flanders Marine Institute, Wandelaarkaai 7, Ostend, 8400, Belgium*
[3]*Wageningen Marine Research, Haringkade 1, IJmuiden, 1976 CP, The Netherlands*
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[5]*Institute of Biology, Leiden University, Sylviusweg 72, Leiden, 2333BE, The Netherlands*

ABSTRACT:
The aquatic world of animals is an acoustic world as sound is the most prominent sensory capacity to extract information about the environment for many aquatic species. Fish can hear particle motion, and a swim bladder potentially adds the additional capacity to sense sound pressure. Combining these capacities allows them to sense direction, distance, spectral content, and detailed temporal patterns. Both sound pressure and particle motion were recorded in a shallow part of the North Sea before and during exposure to a full-scale airgun array from an experimental seismic survey. Distinct amplitude fluctuations and directional patterns in the ambient noise were found to be fluctuating in phase with the tidal cycles and coming from distinct directions. It was speculated that the patterns may be determined by distant sources associated with large rivers and nearby beaches. Sounds of the experimental seismic survey were above the ambient conditions for particle acceleration up to 10 km from the source, at least as detectable for the measurement device, and up to 31 km for the sound pressure. These results and discussion provide a fresh perspective on the auditory world of fishes and a shift in the understanding about potential ranges over which they may have access to biologically relevant cues and be masked by anthropogenic noise.
 https://doi.org/10.1121/10.0006412

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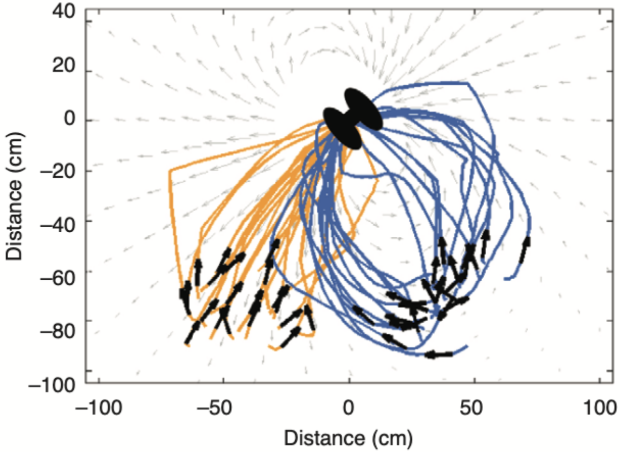
[Editor: Arthur N. Popper] Pages: 2174–2188

Underwater ecology with polarization

Sound-source localization by fish using local acoustic particle motion



[Zeddies, *et al*, 2011]

Take-home message

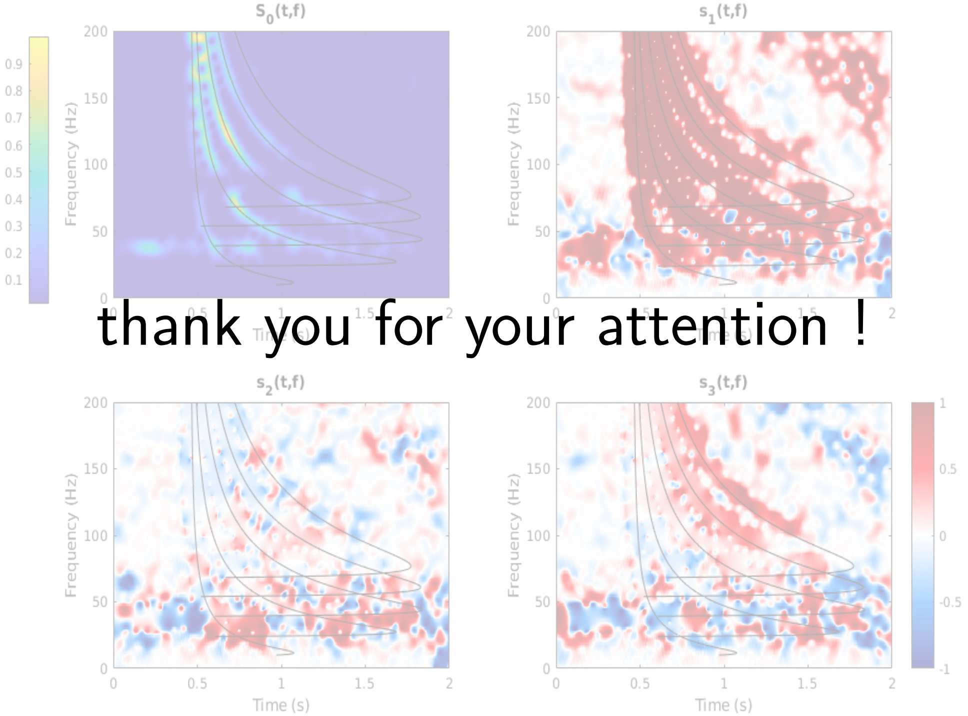
- a general framework for bivariate signals
  polarization physics mathematical guarantees easy to compute
- underwater vector acoustics offers exciting opportunities
  new sensors general case: 3D particle velocity measurements

many open questions and challenges!
signal processing, environment, bioacoustics

Main references

- J. Bonnel, *et al.* (2021)"*Polarization of ocean acoustic normal modes*". The Journal of the Acoustical Society of America, 150(3), 1897-1911.
- J. Flamant, (2018) "*A general approach for the analysis and filtering of bivariate signals*," PhD thesis.

thank you for your attention !



Explicit expression of Stokes parameters

source depth z_s; receiver depth z; range r

complex wavenumber $k_n(f) = k_n^{(r)}(f) - j\beta_n(f)$

$$S_0(f) = \left|\frac{A_n(f,r,z_s)}{2\pi f \rho T}\right|^2 \left[|k_n(f)|^2 \Psi_n(f,z)^2 + \left(\frac{\partial \Psi_n(f,z)}{\partial z}\right)^2\right]$$

$$\frac{S_1(f)}{S_0(f)} = \frac{|k_n(f)|^2 \Psi_n(f,z)^2 - \left(\frac{\partial \Psi_n(f,z)}{\partial z}\right)^2}{|k_n|^2 \Psi_n(f,z)^2 + \left(\frac{\partial \Psi_n(f,z)}{\partial z}\right)^2}$$

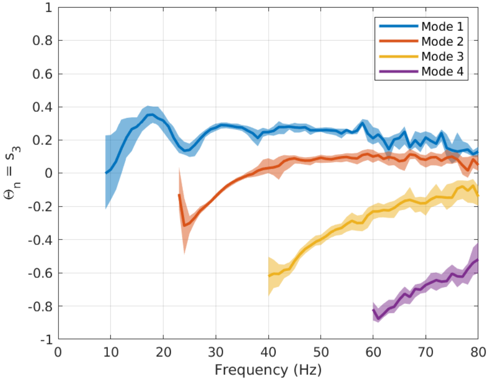
$$\frac{S_2(f)}{S_0(f)} = 2\beta_n(f) \frac{\Psi_n(f,z)\frac{\partial \Psi_n(f,z)}{\partial z}}{|k_n|^2 \Psi_n(f,z)^2 + \left(\frac{\partial \Psi_n(f,z)}{\partial z}\right)^2}$$

$$\frac{S_3(f)}{S_0(f)} = -2k_n^{(r)}(f) \frac{\Psi_n(f,z)\frac{\partial \Psi_n(f,z)}{\partial z}}{|k_n|^2 \Psi_n(f,z)^2 + \left(\frac{\partial \Psi_n(f,z)}{\partial z}\right)^2}.$$

Observing the degree of polarization Φ

Stokes s_1, s_2, s_3 are independent from the source location

variations across experiments $\rightarrow \begin{cases} \text{ambient noise} \\ \text{environemental fluctuations} \end{cases}$



assuming spatial homogeneity, constitute an experimental observation of Φ

Filtering with the degree of polarization Φ

bivariate signal $x(t)$

$=$

polarized $x_{\mathrm{p}}(t)$

$+$

unpolarized $x_{\mathrm{u}}(t)$